An Introduction to Timed Automata

Time is of the essence

Gaëtan Staquet
gaetan.staquet@ec-nantes.fr
https://www.gaetanstaquet.com/research/schools.html

École Centrale de Nantes – LS2N

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General context

Some systems are **critical**.

- A plane ought to not crash.
- Two trains cannot be in the same place at the same time.
- *Ingenuity* (*Perseverance*'s helicopeter) should be able to fly at least 30 seconds every week.

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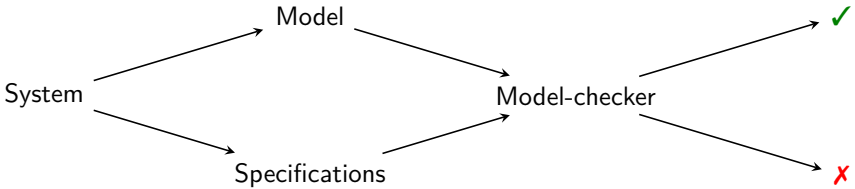
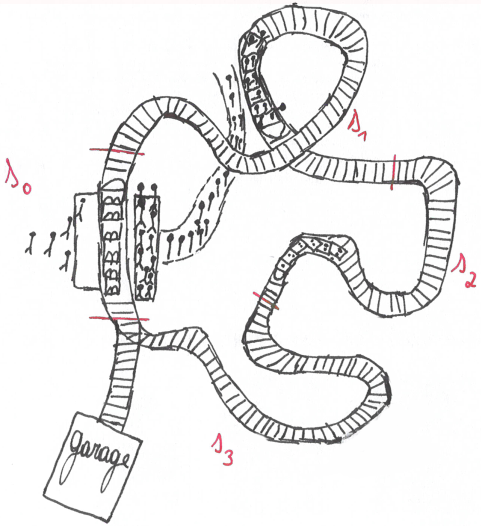
But we may miss some cases...

$\rightsquigarrow$ **Model-checking!**

Model-checking in a nutshell



Our running-example: an attraction



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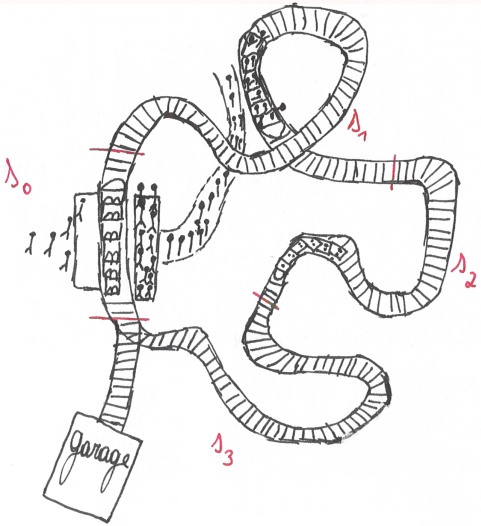

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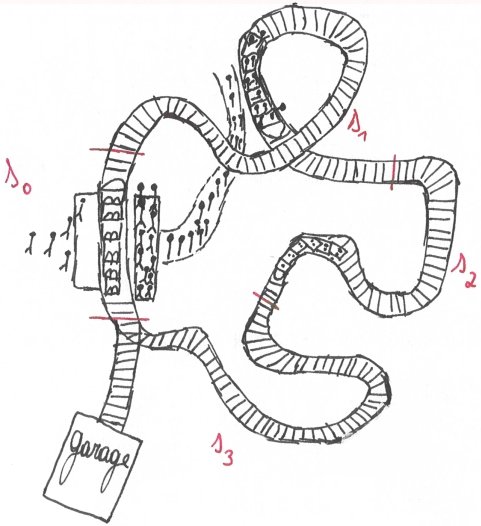

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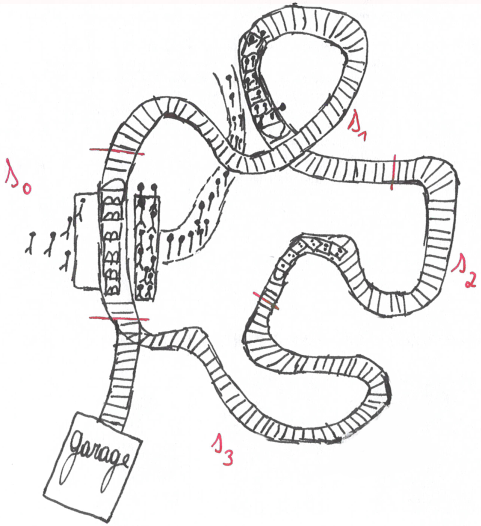

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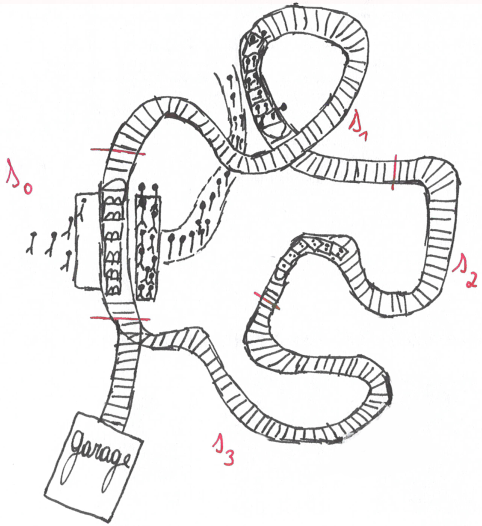

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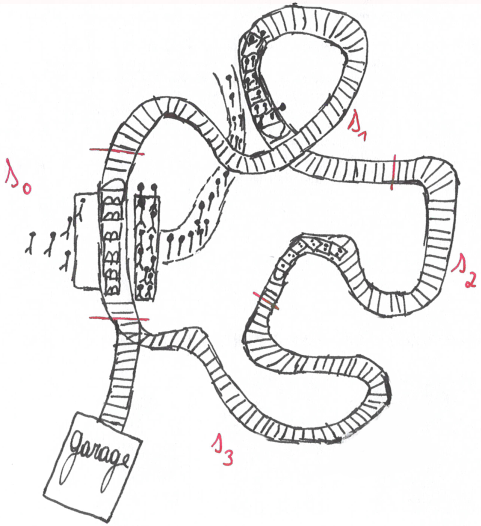
- The **station** manages when a train leaves the garage, and when a train leaves the station.
- All trains share the exact same behavior.
- Timing constraints:
  - Time for a train in s_0: in $(3, 4]$.
  - Time in s_1: 2.
  - Time in s_2: 3.
  - Time in s_3: in $(2, 5]$.

Our running example: properties



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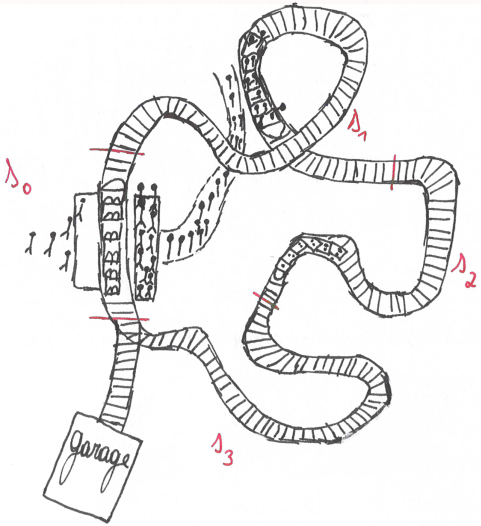

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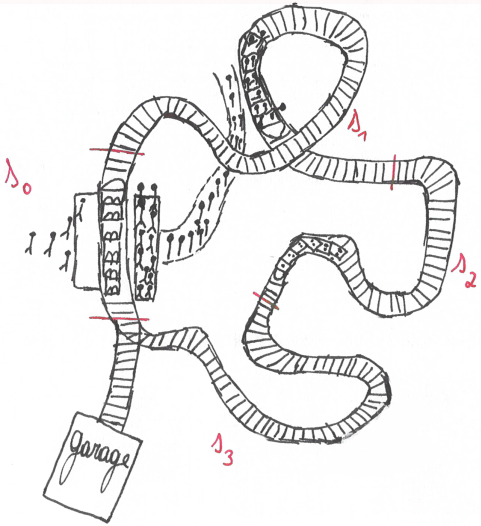

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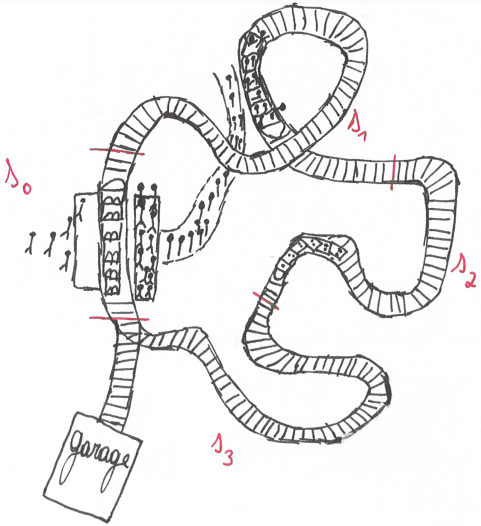

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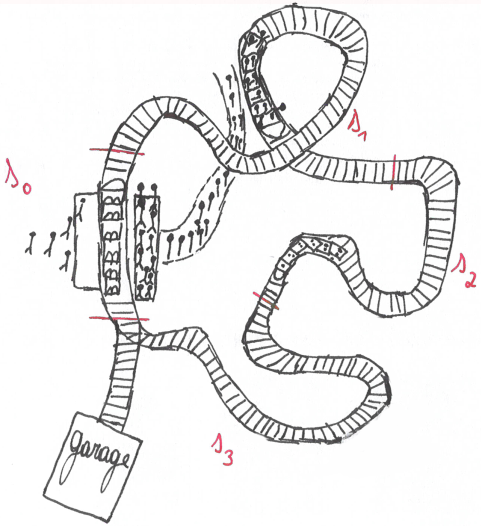

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Symbols, alphabets, and words

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- For an alphabet Σ, we write Σ^* for the set of all words over Σ.

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- $aab0b$ is a word over $\{a, b, 0\}$; $1cc1$ is a word over $\{c, 1\}$.
- $\{a, b\}^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, \dots\}$.

A first family of models: finite automata

Definition 1 (Finite automata). A **finite automaton** (**FA**) is a tuple $\mathcal{A} = (\Sigma, Q, q_0, \delta)$ with

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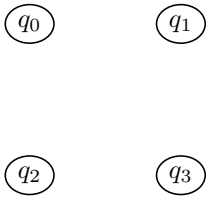

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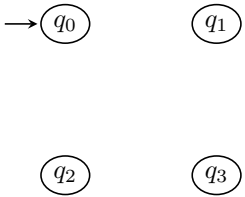

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- Σ an **alphabet**;
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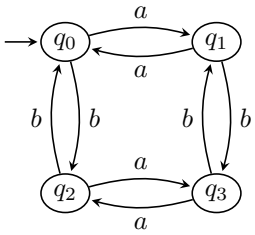
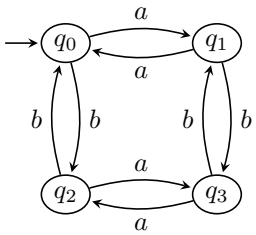
- $\delta : Q \times \Sigma \to \mathcal{P}(Q)$ the **transition function** (or transition relation).



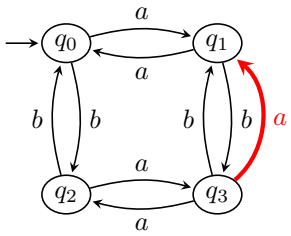
Finite automata and runs

- $q_0 \xrightarrow{a} q_1$ is a **transition** of the FA because $\delta(q_0, a) \ni \{q_1\}$
- A **run** is a finite sequence of transitions: $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_3 \xrightarrow{a} \xrightarrow{a} q_3$ is a run.
- We write $q_0 \xrightarrow{abaa} q_3$ as a shorthand.



Nondeterministic transitions

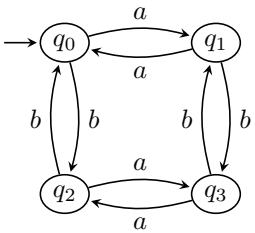
- From q_3, there are two choices when reading a: $q_3 \xrightarrow{a} q_2$ or $q_3 \xrightarrow{a} q_1$.
- So, $q_0 \xrightarrow{abaa} q_3$ or $q_0 \xrightarrow{abaa} q_0$ are two valid runs, reading the same word.
- We say that the transition and the FA are **nondeterministic**.
- Examples and exercices here will be over deterministic automata, but keep in mind that nondeterminism can appear.



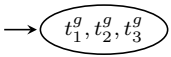
Finite automata and properties

Giving properties over the system $\Leftrightarrow$ giving properties over the runs.

- For every run over a word w, the run ends in q_1 if and only if the number of a in w is odd and the number of b is even.
- Every run can be continued, *i.e.*, we do not have a **deadlock**.



A first model of our attraction



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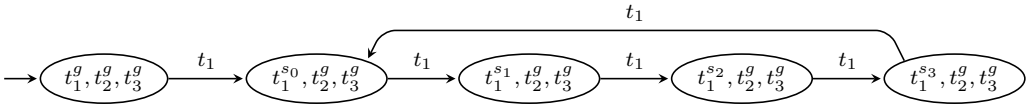

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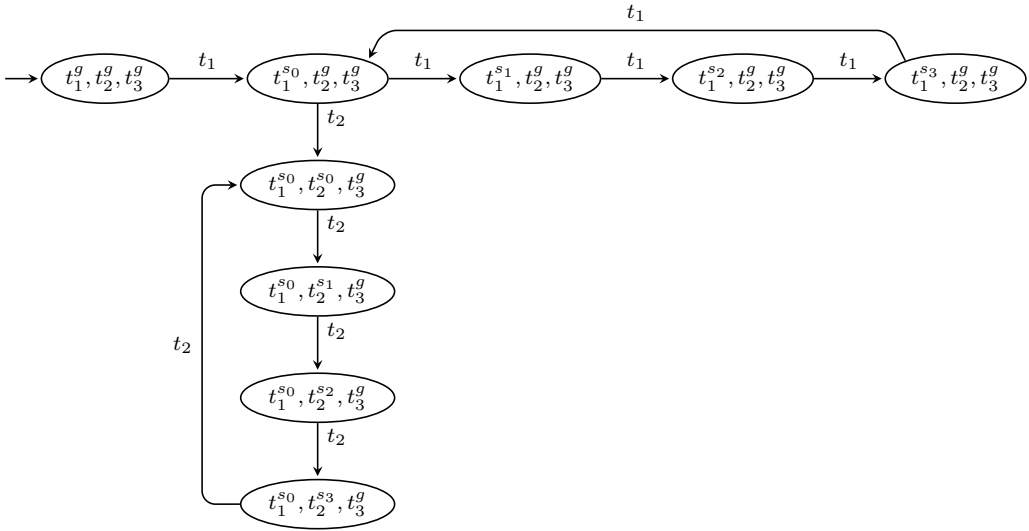

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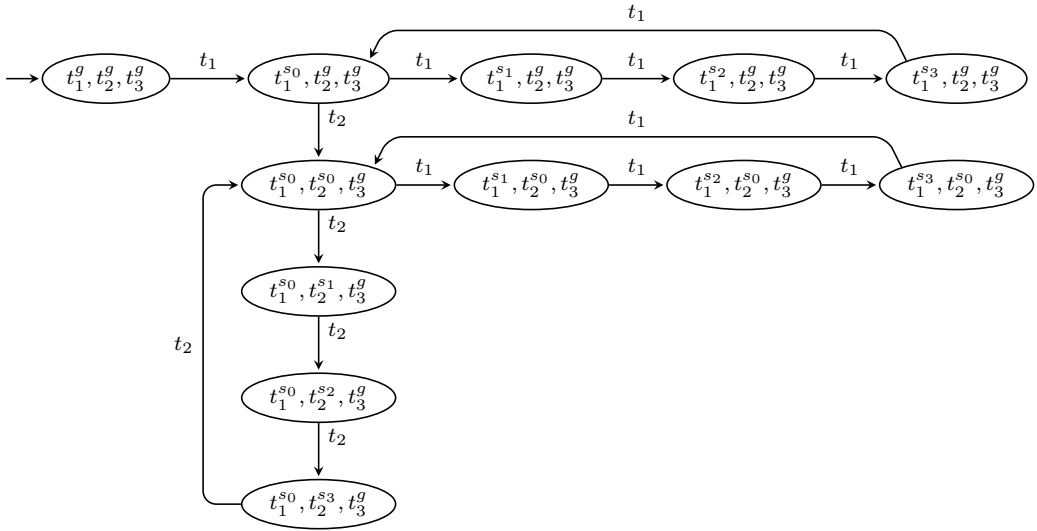

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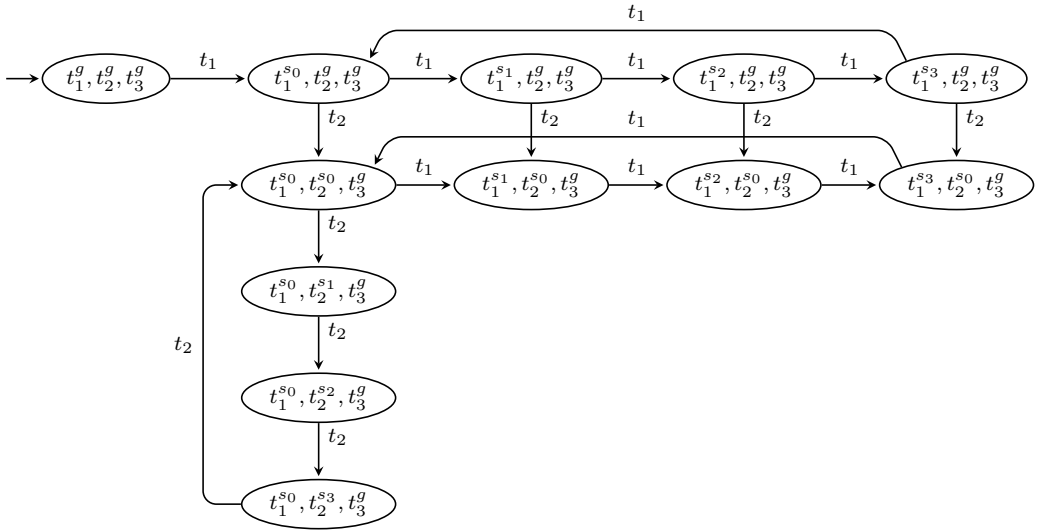

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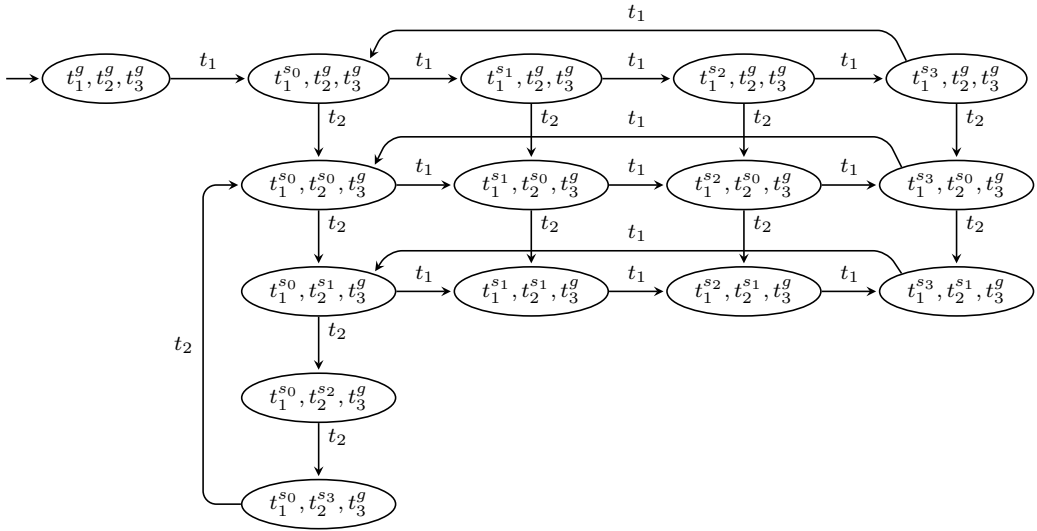

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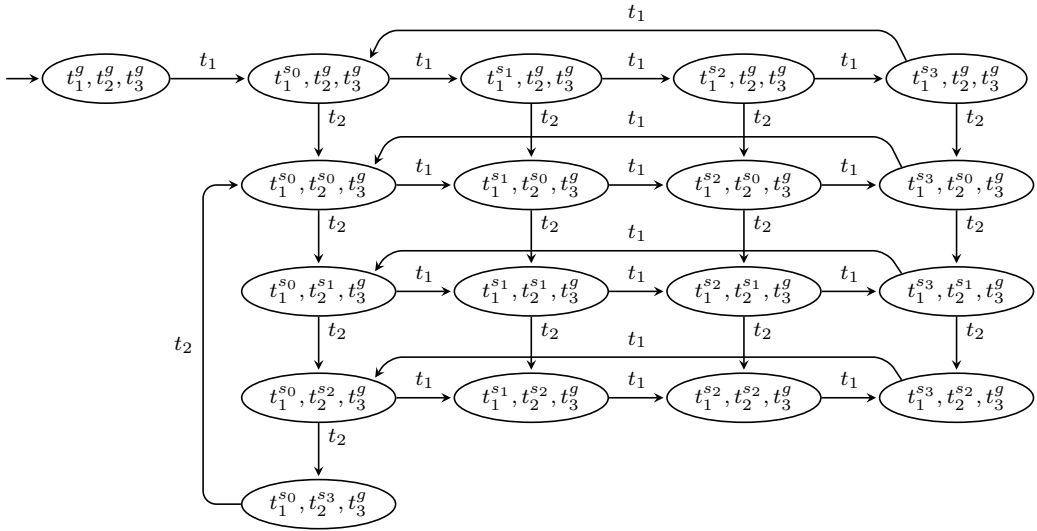

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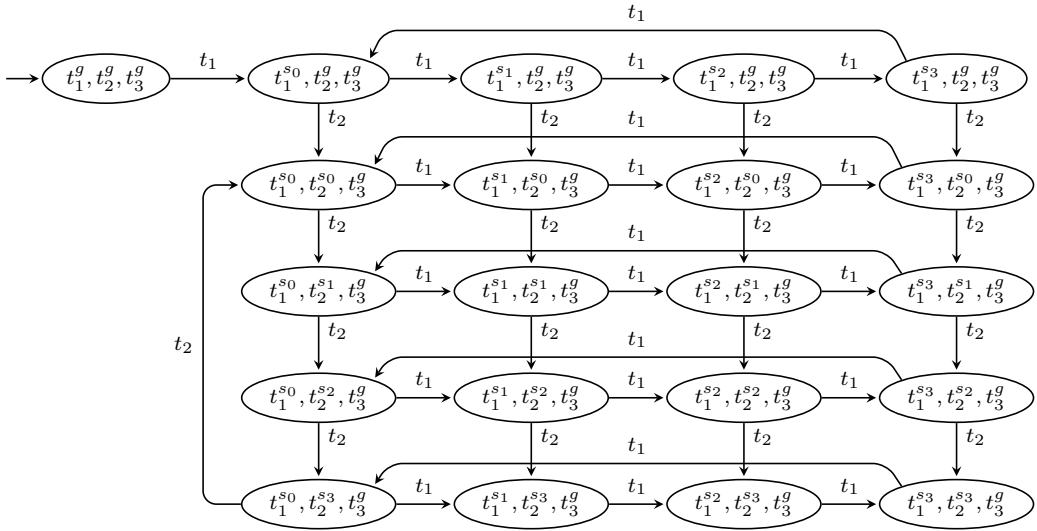

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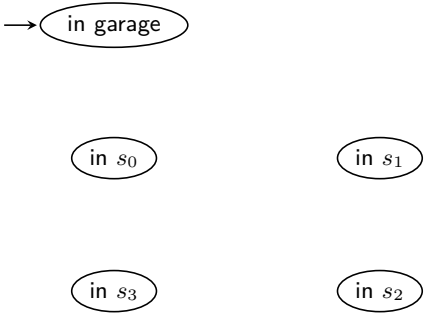
Creating a single big automaton for the whole system is not a good idea.

Let's exploit **compositionality**!

Station and trains

Model for the **station**.



Model for the **train** t_i.

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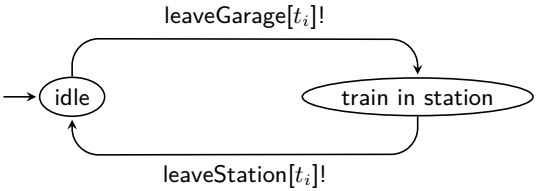

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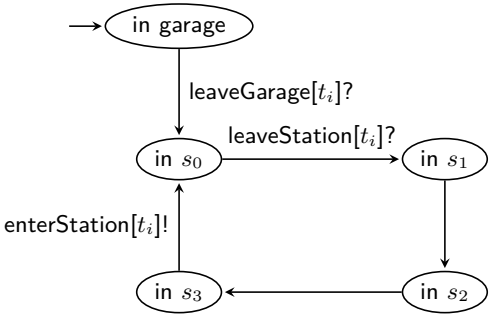

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- $s!$ sends the symbol s;
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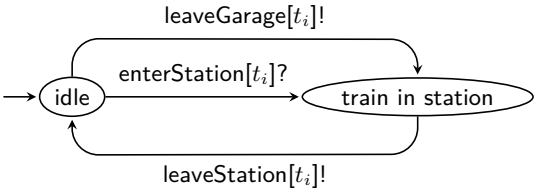

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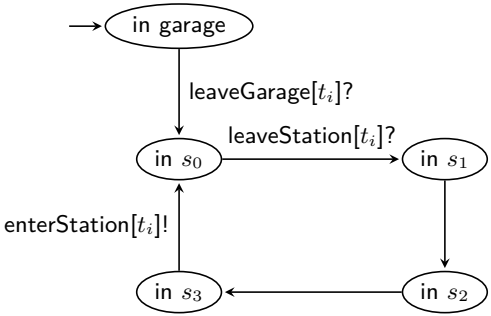

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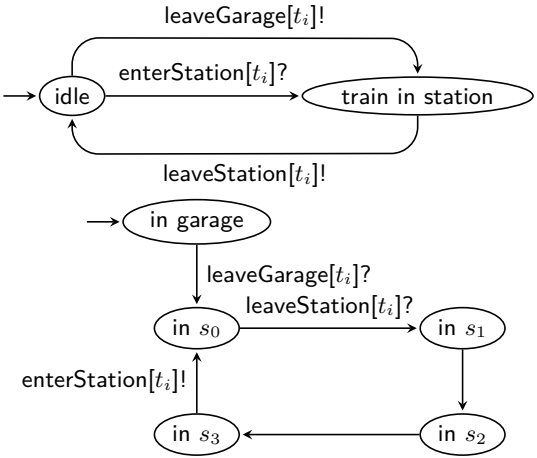

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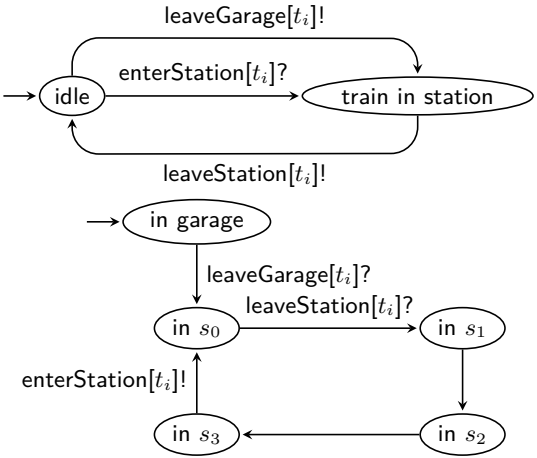

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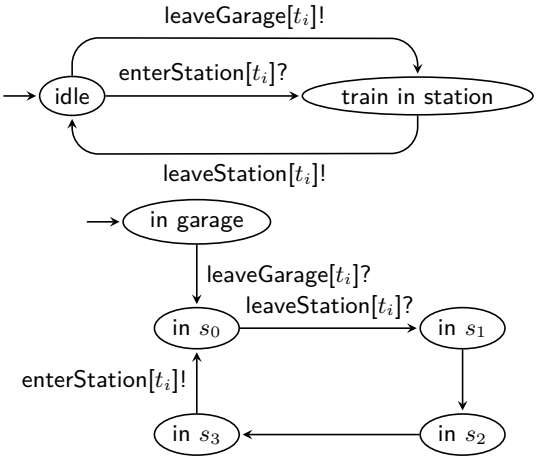

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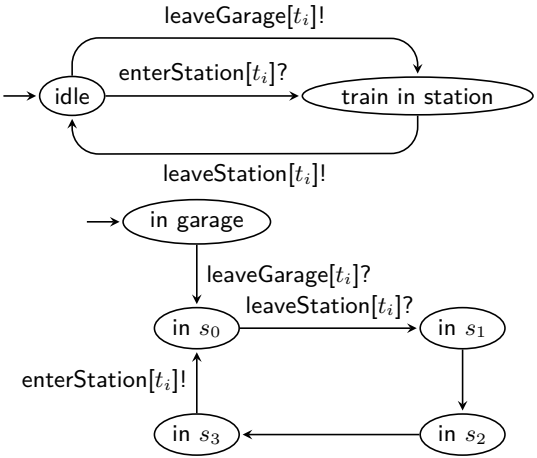

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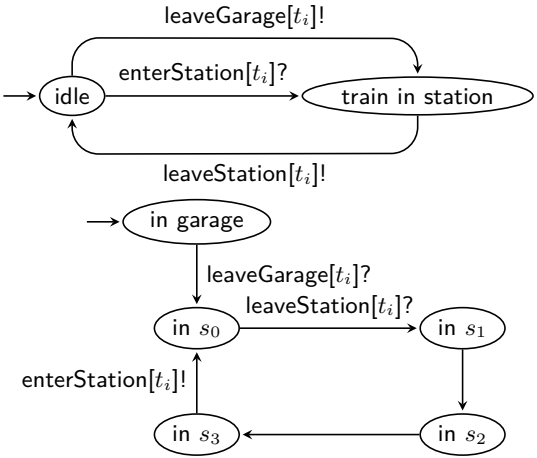

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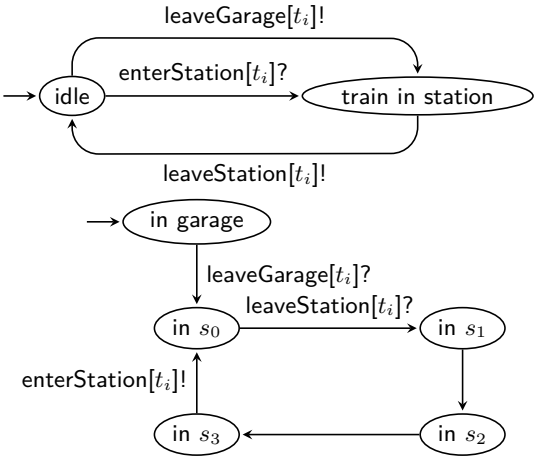

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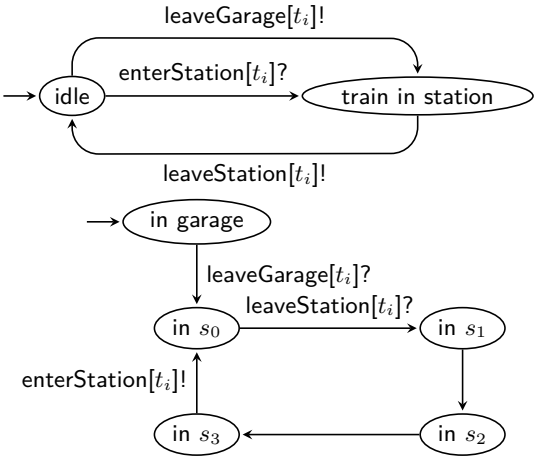

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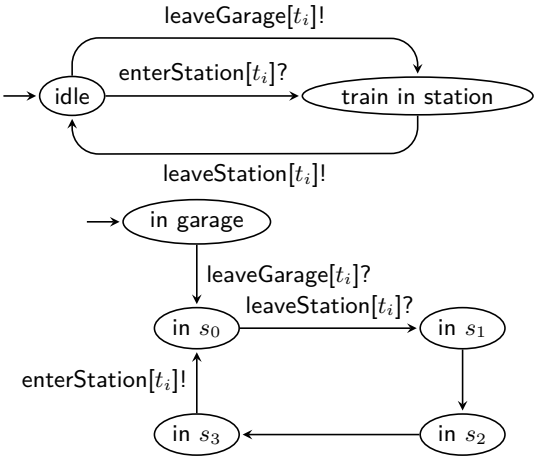

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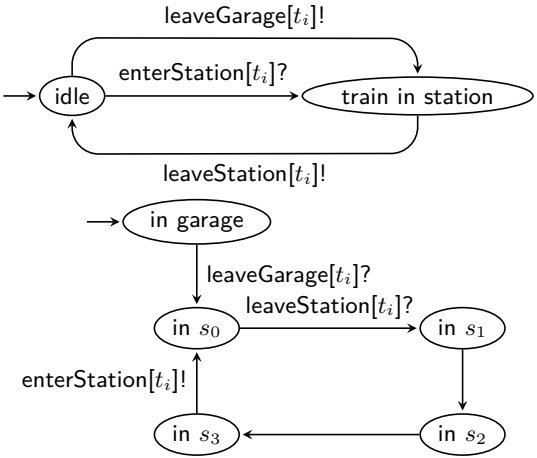

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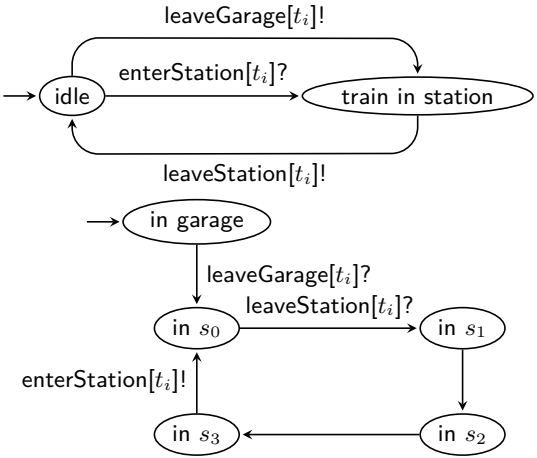

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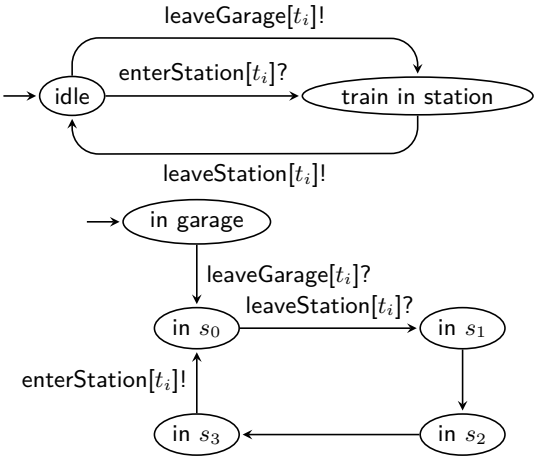
There is still hope



- There is a part of the system's specifications we ignored so far!
- Timing constraints:
  - Time for a train in s_0: in $(3, 4]$.
  - Time in s_1: 2.
  - Time in s_2: 3.
  - Time in s_3: in $(2, 5]$.
- Finite automata cannot handle time...

Clocks

Let's extend finite automata with a way to take time into account.
Definitions for this section come from Bouyer et al., "Model Checking Real-Time Systems", 2018.

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Definition 2 (Clocks and valuations). A **clock** x is a real-valued variable.
Let C be a set of clocks. A **(clock) valuation** over C is a mapping $\nu : C \to \mathbb{R}^{\geq 0}$, which assigns to each clock a real value.
We write $\mathsf{Val}(C)$ for the set of all valuations over C.
We write $\mathbf{0}_C$ for the valuation assigning 0 to every clock $x \in C$.

Example 3. Let $C = \{x, y\}$.

- $(x = 5, y = 1)$ is a valuation over C.
- $\mathsf{Val}(C) = \{x = k \mid k \in \mathbb{R}^{\geq 0}\} \times \{y = k \mid k \in \mathbb{R}^{\geq 0}\}$.

Clock operations

Operations. A clock x will be used to remember how many **time units elapsed** since the last time x was **set to zero**.

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Definition 4 (Clock operations). Let C be a set of clocks and $\nu \in \mathrm{Val}(C)$ be a valuation over C.

Elapsing time For every $t \in \mathbb{R}^{\geq 0}$, the valuation $\nu + t$ is defined by

$$\forall x \in C : (\nu + t)(x) = \nu(x) + t.$$

Resetting to zero For every $R \subseteq C$, the valuation $\nu[R]$ is defined by

$$\forall x \in C : \nu[R](x) = \begin{cases} 0 & \text{if } x \in R \\ \nu(x) & \text{if } x \in C \setminus R \end{cases}$$

Clock constraints

Comparisons. We will need to enable or disable behaviors, according to the current clock valuation.

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Definition 5 (Clock constraints). Let C be a set of clocks. The set $\Phi(C)$ of **clock constraints** over C is defined by

$$\Phi(C) \ni \varphi ::= true \mid x \bowtie k \mid \varphi_1 \wedge \varphi_2 \qquad (x \in C, k \in \mathbb{N}, \bowtie \in \{<, \leq, =, \geq, >\}).$$

Let $\nu \in \mathsf{Val}(C)$ and $\varphi \in \Phi(C)$. We write $\nu \models \varphi$ if

- $\varphi = true$;
- when $\varphi = x \bowtie k$, $\nu(x) \bowtie k$; or
- when $\varphi = \varphi_1 \wedge \varphi_2$, $\nu \models \varphi_1 \wedge \nu \models \varphi_2$.

Example 6. Let $C = \{x, y\}$ and $\varphi = (x > 4) \wedge (y \leq 3)$. Then,

- $(x = 4, y = \frac{4}{3}) \not\models \varphi$, as $(x = 4, y = \frac{4}{3}) \not\models (x > 4)$.
- $(x = 4.5, y = \frac{10}{3}) \not\models \varphi$, as $(x = 4.5, y = \frac{10}{3}) \not\models (y \leq 3)$
- $(x = 4.5, y = \frac{4}{3}) \models \varphi$.

Timed automata

Definition 7 (Timed automata). A **timed automaton** (**TA**) is a tuple $\mathcal{A} = (\Sigma, Q, q_0, C, I, \delta)$ with

- Σ an **alphabet**;
- Q a non-empty finite set of **states**;
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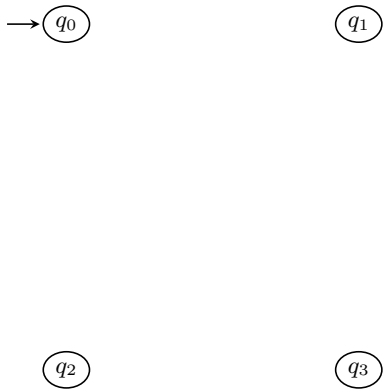

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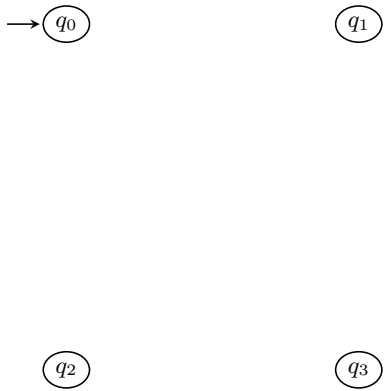

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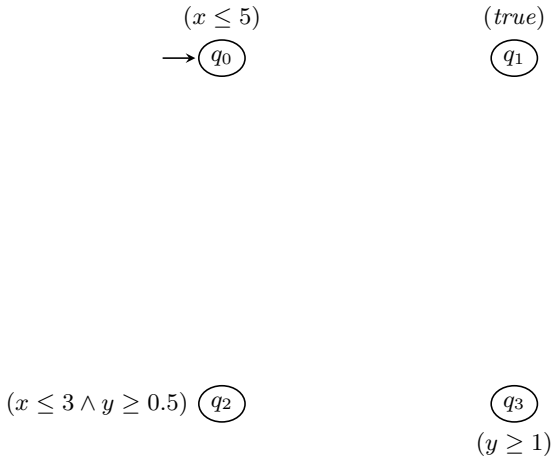

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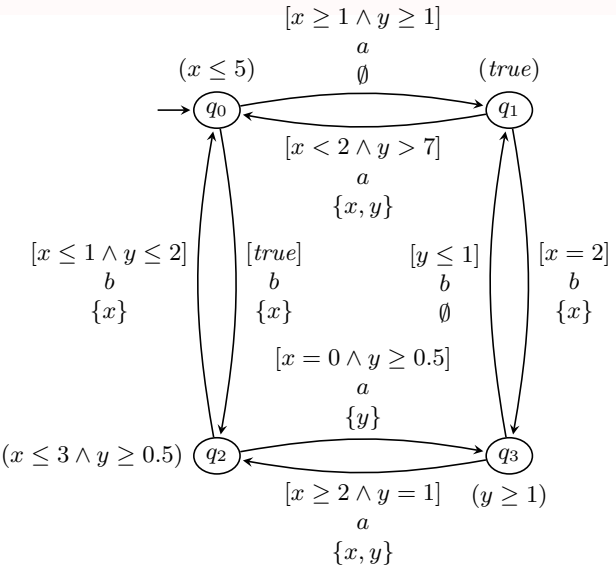
- $\delta : Q \times \Phi(C) \times \Sigma \to \mathcal{P}(Q \times \mathcal{P}(C))$ the **transition function**.



Configurations, transitions, and runs

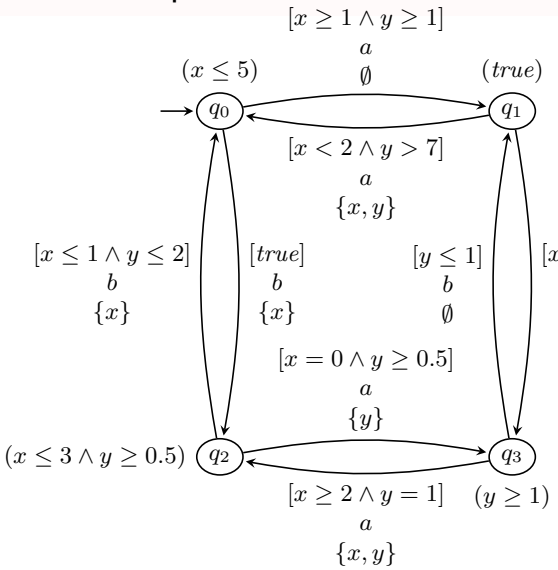
Definition 8 (Configurations and transitions). Let $\mathcal{A} = (\Sigma, Q, q_0, C, I, \delta)$ be a TA.

- A **configuration** is a pair $(q, \nu) \in Q \times \mathsf{Val}(C)$.
- From $(q, \nu) \in Q \times \mathsf{Val}(C)$, there are two possible types of **transitions**.

  Delay With $t \in \mathbb{R}$, $(q, \nu) \xrightarrow{t} (q, \nu + t)$ if and only if $\forall t' \in [0, t] : v + t' \models I(q)$.

  Discrete With $a \in \Sigma$, $(q, \nu) \xrightarrow{\varphi, a, R} (q', \nu[R])$ if and only if
  - $\delta(q, \varphi, a) \ni (q', R)$; and
  - $\nu \models \varphi$.
- The **initial configuration** is $(q_0, \mathbf{0}_C)$.
- A **run** is a finite sequence of delay and discrete transitions, starting from the initial configuration.

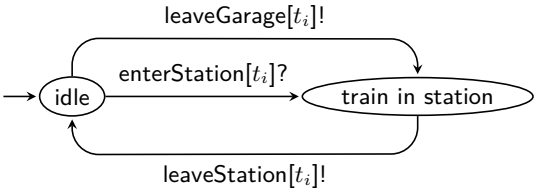
Runs – Example



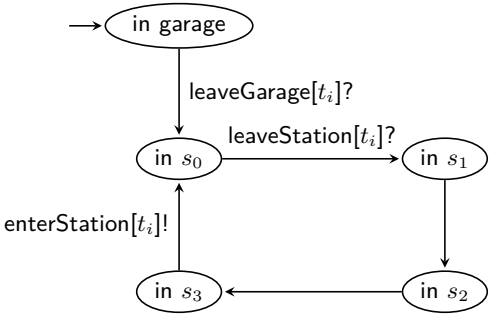
$$(q_0, (x = 0, y = 0)) \xrightarrow{2} (q_0, (x = 2, y = 2))$$
$$\xrightarrow{a} (q_1, (x = 2, y = 2))$$
$$\xrightarrow{b} (q_3, (x = 0, y = 2))$$
$$\xrightarrow{1} (q_3, (x = 1, y = 3))$$

From there, all we can do is wait. Forever...

Previously, in our roller coaster



Model for the **station**.



Model for the **train** t_i.

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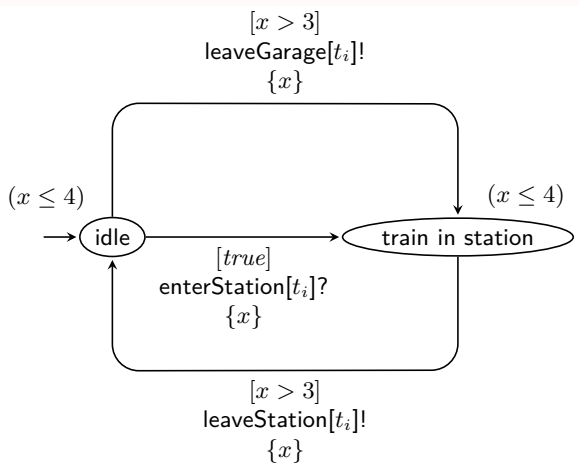
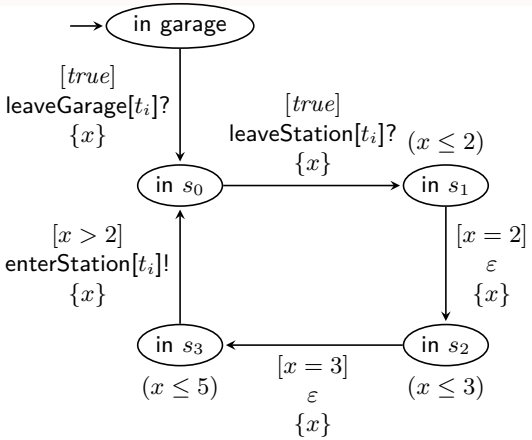
Timed roller coaster

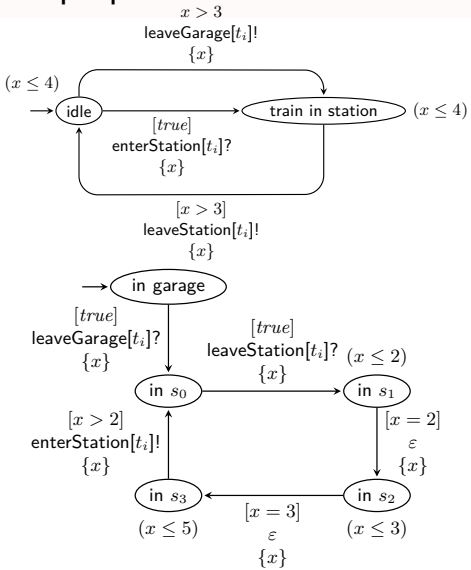


Model for the **station** with clock x.



Model for the **train** t_i with clock x.

Which properties hold?



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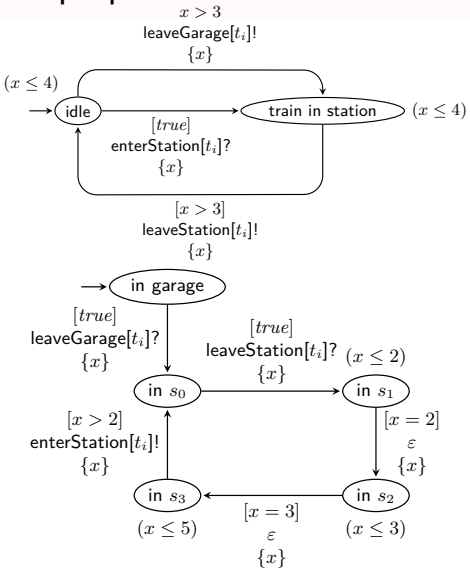

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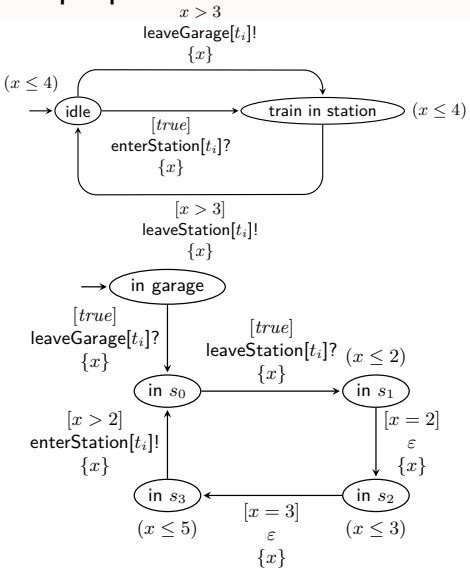

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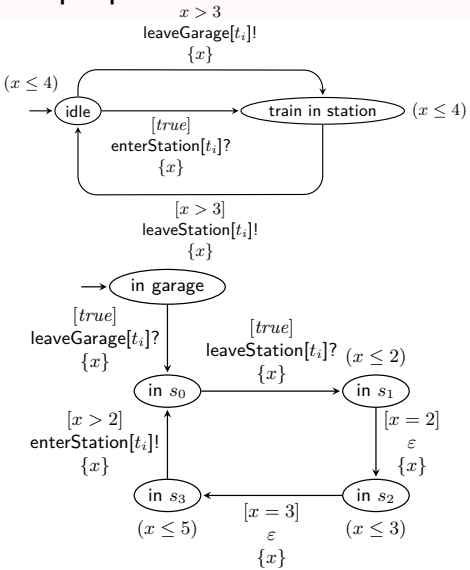

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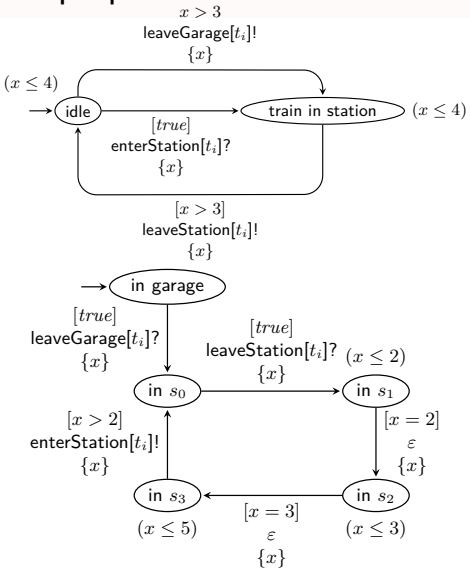

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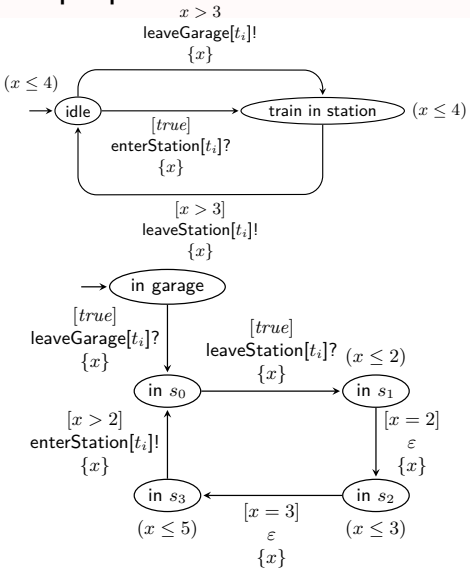

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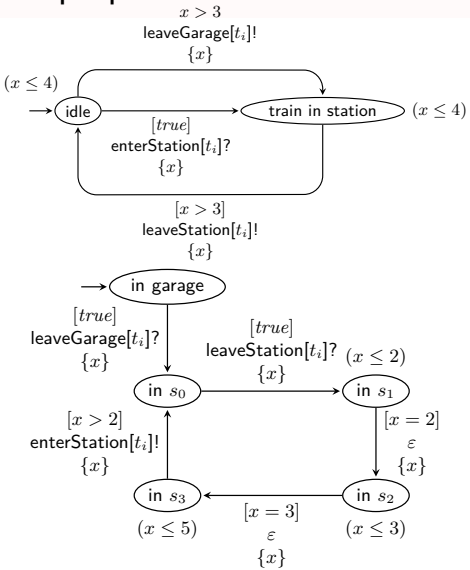

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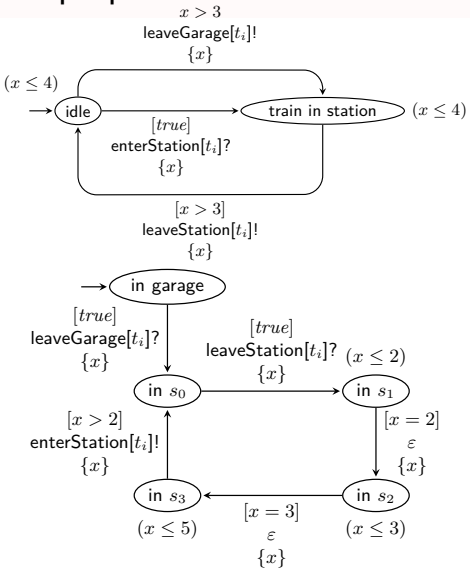

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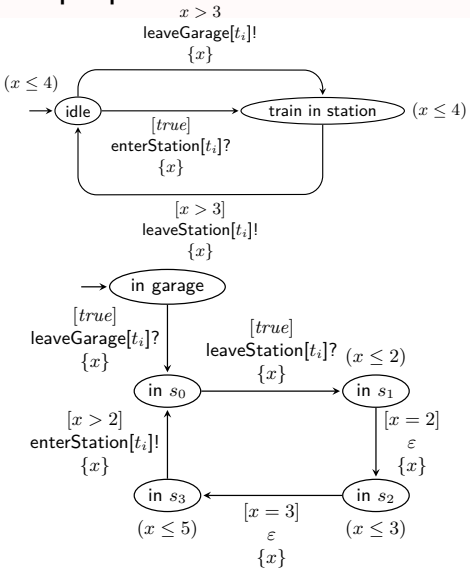

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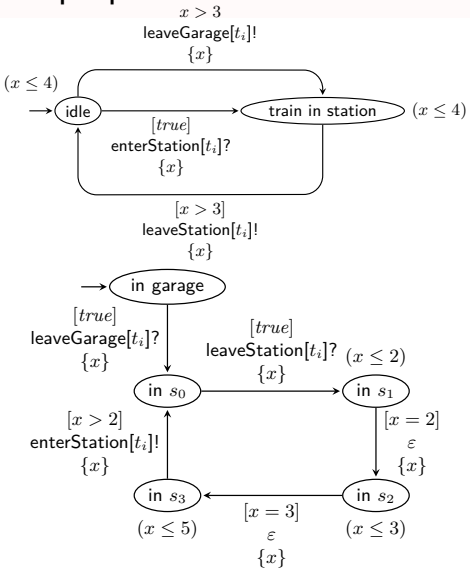

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Two kinds of properties

In this course and in the exercise session, we focus on two types of properties.

- **Reachability**: is there an execution of the system that reaches a good state?

> *Example 9.* It must be possible to have one train running.

- **Safety**: the system can never reach a bad state.

> *Example 10.* Two trains can never be in the same section at the time.
> A train must always be able to move forward.

Here, we give the intuition for checking reachability.

Not everything is reachable

Main observation. Given a TA $\mathcal{A}$ and a configuration (q, ν), it is not always possible to construct a run from $(q_0, \mathbf{0}_C)$ to (q, ν).

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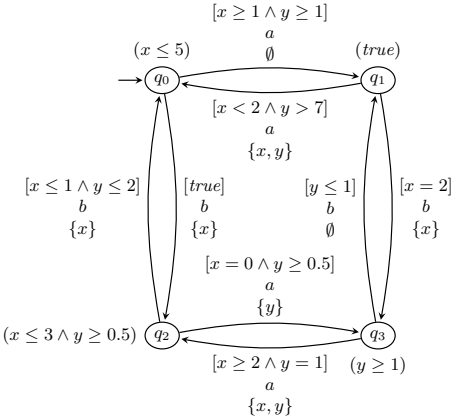

We can reach $(q_1, (x = 1, y = 1))$:

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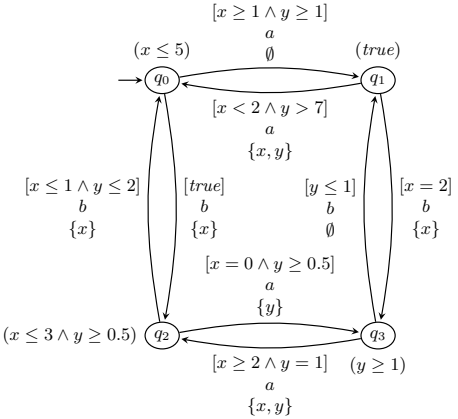

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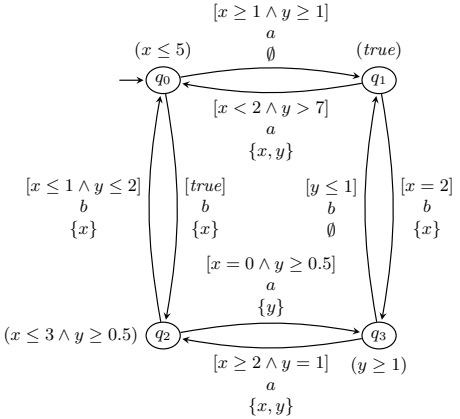

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We cannot reach $(q_0, (x = 0, y = 10))$. That's hard to see...

Constructing the reachable fragment – Rough idea

- We know that we start from $(q_0, \mathbf{0}_C)$.

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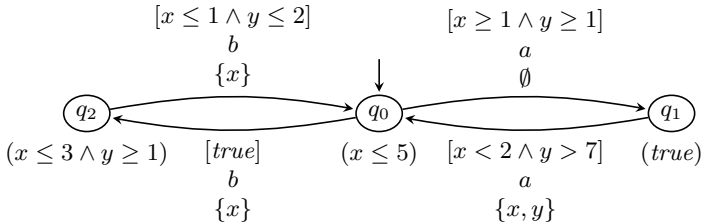
So, along an exploration of the TA, we can accumulate **constraints** over the valuations.

Constructing the reachable fragment – Example



$\longrightarrow \boxed{q_0, x = y = 0}$

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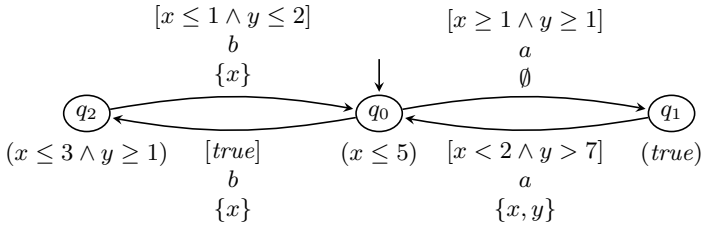

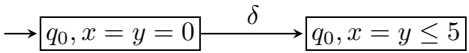

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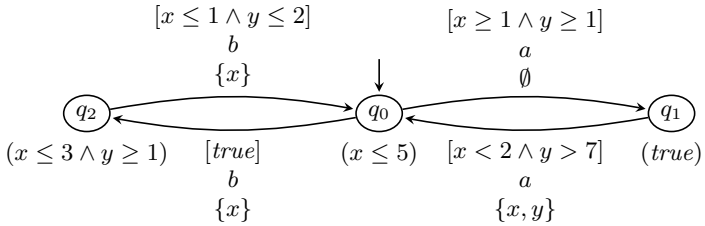
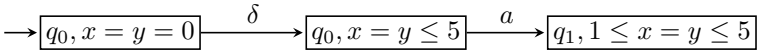
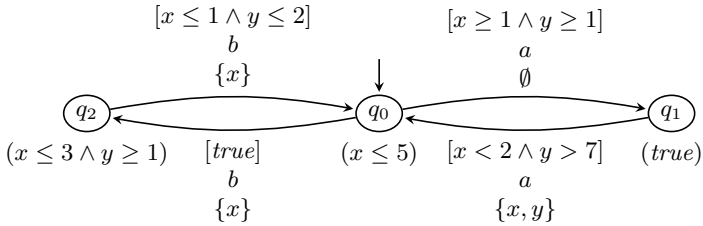


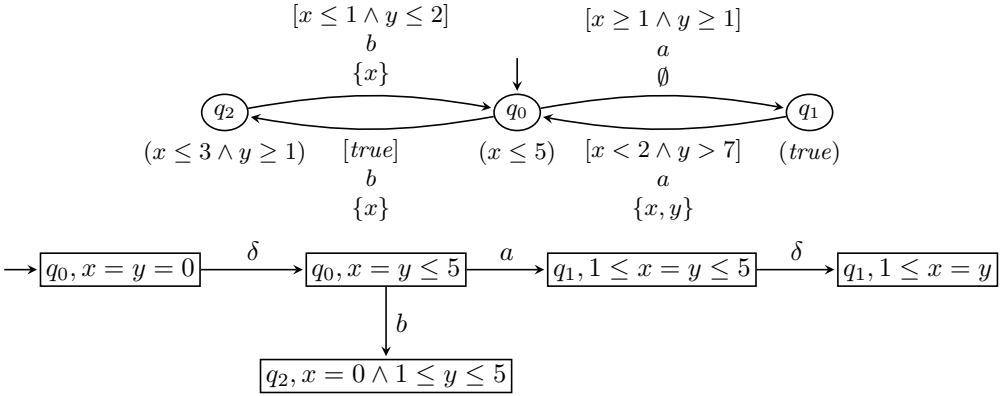


Constructing the reachable fragment – Example

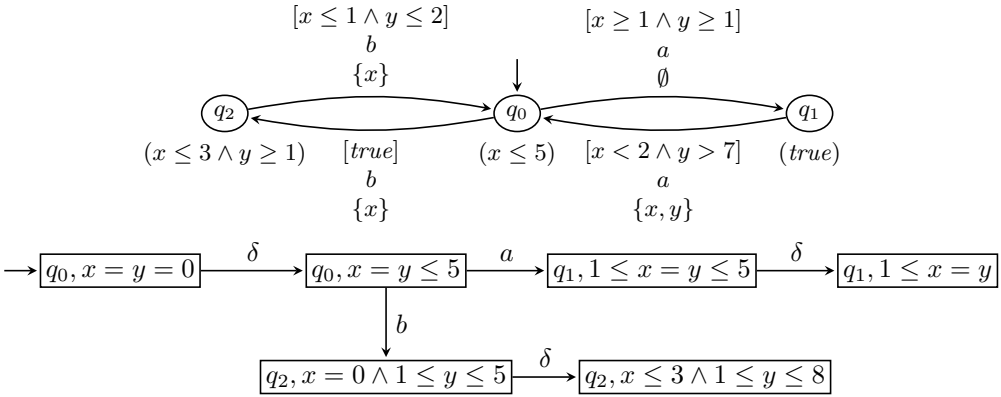


$$\rightarrow \boxed{q_0, x = y = 0} \xrightarrow{\delta} \boxed{q_0, x = y \leq 5} \xrightarrow{a} \boxed{q_1, 1 \leq x = y \leq 5} \xrightarrow{\delta} \boxed{q_1, 1 \leq x = y}$$

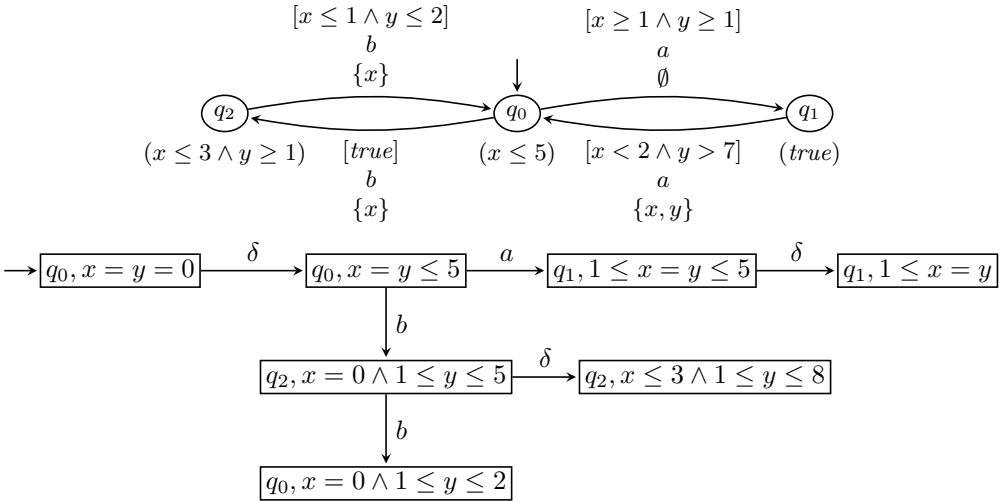
Constructing the reachable fragment – Example



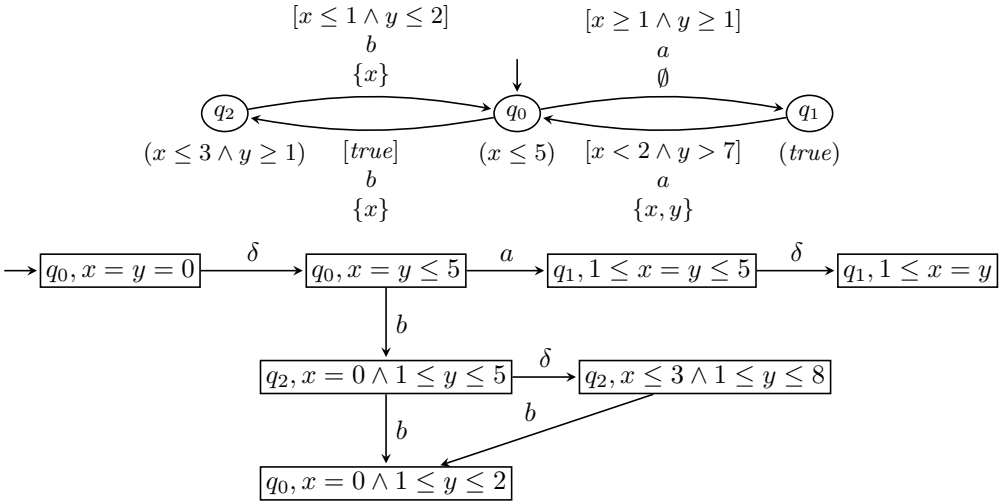
Constructing the reachable fragment – Example



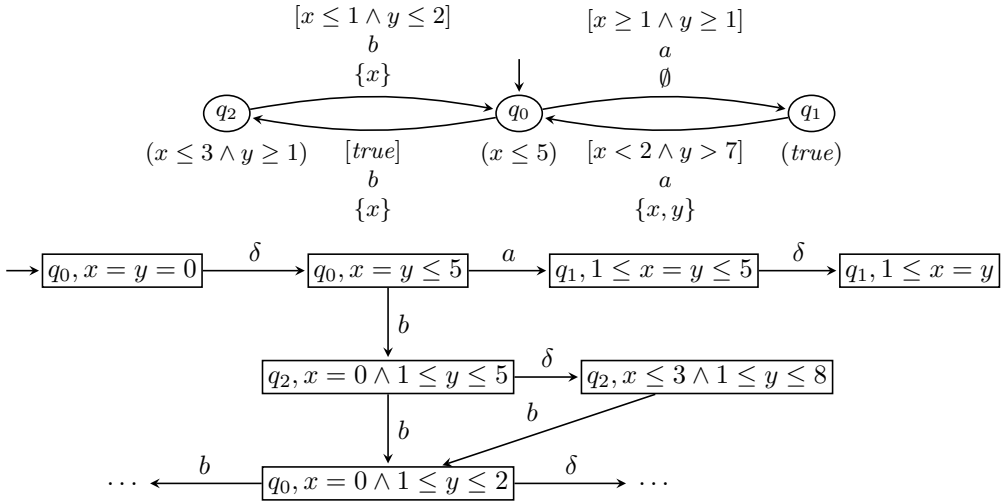
Constructing the reachable fragment – Example



Constructing the reachable fragment – Example



Constructing the reachable fragment – Example



Deciding reachability properties

Theorem 11. Let $\mathcal{A}$ be a TA, and (q, ν) be a configuration. Deciding wether (q, ν) is reachable is PSPACE-complete.

Tools

Luckily, we don't have to verify the properties by hand!

- TChecker: open-source, no graphical interface, timed automata.
  `https://github.com/ticktac-project/tchecker`
- IMITATOR: open-source, no graphical interface, **parametric** timed automata.
  `https://www.imitator.fr/`
- Roméo: open-source, has a graphical interface, timed **Petri nets**.
  `https://romeo.ls2n.fr/`
- **UPPAAL**: closed-source, has a graphical interface, timed automata.
  `https://uppaal.org/`

After the break

In the practical session:

- To get started with **UPPAAL**: the roller coaster example.
- An exercise on modeling a system that shares resources between processes.
- An exercise to write properties for a model of an amusement park.